Suggested Solutions to:
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Industrial Organization
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Question 1: Collusion between Stackelberg competitors

(a) Solve for the subgame perfect Nash equilibrium of the Stackelberg game described above.

To solve for the subgame perfect Nash equilibrium, we can use backward induction. We thus first solve firm 2's problem of choosing q_2 optimally, for any given q_1. Thereafter, we plug the resulting expression for the optimal q_2 (as a function of q_1) into firm 1's profit function and then maximize this with respect to q_1.

Firm 2's problem is to maximize

$$\pi_2 = (2 - c_2 - q_1 - q_2)q_2$$

with respect to q_2, while taking q_1 as given. The first-order condition can be written as

$$\frac{\partial \pi_2}{\partial q_2} = 2 - c_2 - q_1 - 2q_2 \leq 0, \tag{1}$$

which holds with equality if the optimal q_2 is strictly positive. From (1) it follows that we can write firm 2's best reply as

$$R_2(q_1) = \begin{cases} \frac{2-c_2-q_1}{2} & \text{if } q_1 \leq 2 - c_2 \\ 0 & \text{if } q_1 > 2 - c_2. \end{cases} \tag{2}$$

Let us here derive an equilibrium in which $q_1 \leq 2 - c_2$, meaning that firm 2 chooses to be active and its best reply is given by the first row above. For a large enough cost advantage for firm 1, we should expect that an equilibrium where firm 2 is not active exists. This possibility can be studied if we want to be more ambitious with our analysis. Plugging the top row of (2) into firm 1's profit yields

$$\pi_1 = \left[2 - c_1 - q_1 - \frac{2 - c_2 - q_1}{2}\right] q_1 = \left(1 - c_1 + \frac{c_2}{2} - \frac{q_1}{2}\right) q_1.$$

The first-order condition yields

$$\frac{\partial \pi_1}{\partial q_1} = 1 - c_1 + \frac{c_2}{2} - q_1 = 0 \Leftrightarrow q_1 = \frac{2 - 2c_1 + c_2}{2}.$$

Plugging this back into (2), we also have

$$q_2 = \frac{2 + 2c_1 - 3c_2}{4}. \tag{3}$$

Summing up:

In the SPNE, firm 1 chooses

$$q_1 = \frac{2 - 2c_1 + c_2}{2},$$

and firm 2 chooses q_2 accrding to (2). The equilibrium outcome is given by

$$(q_1, q_2) = \left(\frac{2 - 2c_1 + c_2}{2}, \frac{2 + 2c_1 - 3c_2}{4} \right).$$

(b) For what values of c_1 and c_2 is the outcome of the equilibrium that you found in part (a) Pareto efficient? Prove your answer formally.

This question is a very close parallel to a question in this year's regular exam. The same intuition applies and the same approach can be used to prove that the equilibrium outcome is never Pareto optimal.

In short, we should intuitively expect that the equilibrium outcome of the Stackelberg game is not Pareto efficient. The firms are not coordinating their output decisions and they fail to internalize the negative externality that their output decisions have on the rival's profit. The firms should both gain if they jointly contracted their output levels at least somewhat.

We can formalize this intuition by imagining that each firm produces its equilibrium quantity in the Stackelberg game *minus* some small but strictly positive constant ϵ. We then write up profit expressions for the firms and show (e.g., by differentiating) that each firm's profit becomes larger as we increase ϵ, for small enough but positive values of ϵ.

(c) Investigate under what conditions the two firms' following the above trigger strategy constitutes a subgame perfect Nash equilibrium of the infinitely repeated game. In particular, derive a (necessary and sufficient) condition for firm i (for $i = 1, 2$) not to have an incentive to deviate from the strategy (given that the other firm follows it). Each condition should be stated as $\delta \geq K_i$, where K_i is function only of λ.

We need to check two kinds of possible deviations from the specified strategy:

- No firm must have an incentive to deviate unilaterally along the equilibrium path. (This is a requirement for having a Nash equilibrium.)
- No firm must have an incentive to deviate unilaterally off the equilibrium path. (This is a requirement for subgame perfection.)

The requirement in the second bullet point is, by standard arguments (see lecture slides), unproblematic.

Consider the requirement in the first bullet point.

- Under collusion, the firms play $q_1 = 1 - \lambda$ and $q_2 = \lambda$. Therefore, firm 1's per-period payoff equals

  $$\pi_1^c = (2 - q_1^c - q_2^c)q_1^c = 1 - \lambda.$$

  And firm 2's per-period payoff equals

  $$\pi_2^c = (2 - q_1 - q_2)q_2 = \lambda.$$

- If firm 1 were to deviate, the deviation would be observed by firm 2 in the same period, before the profit levels for that period have been realized; moreover, if firm 2 follows the trigger strategy, it responds to the deviation by playing q_2^S. Therefore, the optimal deviation for firm 1 is simply

q_2. This means that the deviation quantities are the same as the punishment quantities. The condition for firm 1 not having an incentive to deviate can thus be written as

$$\pi_1^c \geq \pi_1^S \Leftrightarrow 1 - \lambda \geq \frac{1}{2} \Leftrightarrow \lambda \leq \frac{1}{2},$$

which always holds under the assumptions in the model description.

- Next consider the incentives of firm 2 to deviate from the trigger strategy. If not deviating, firm 2's present-discounted sum of profits equals

  $$V_2^c = \pi_2^c + \delta\pi_2^c + \delta^2\pi_2^c + \cdots = \frac{\pi_2^c}{1-\delta} = \frac{\lambda}{1-\delta}.$$

  If firm 2 deviates in a particular period, then firm 1 will immediately after, in the following period, observe the deviation and revert to the one-shot Stackelberg quantity. The optimal deviation q_2^d therefore maximizes $[2 - (1-\lambda) - q_2]\, q_2$ w.r.t. q_2, which means that $q_2^d = (1+\lambda)/2$ and that the optimized deviation profits are given by $\pi_2^d = (1+\lambda)^2/4$. A deviation to the optimal deviation strategy q_2^d will give rise to the following present-discounted sum of profits for firm 2:

  $$V_2^d = \pi_2^d + \delta\pi_2^S + +\delta^2\pi_2^S + \cdots = \pi_2^d + \frac{\delta\pi_2^S}{1-\delta} = \frac{(1+\lambda)^2}{4} + \frac{\delta}{4(1-\delta)} = \frac{\delta + (1-\delta)\,(1+\lambda)^2}{4(1-\delta)}.$$

  Thus, firm 2 does not have an incentive to deviate if and only if

  $$V_2^c \geq V_2^d \Leftrightarrow \frac{\lambda}{1-\delta} \geq \frac{\delta + (1-\delta)\,(1+\lambda)^2}{4(1-\delta)} \Leftrightarrow \delta \geq \frac{(1-\lambda)^2}{\lambda(2+\lambda)}.$$

We can thus conclude that the specified grim trigger strategy is an SPNE if, and only if,

$$\delta \geq \frac{(1-\lambda)^2}{\lambda(2+\lambda)}.$$

Question 2: Price discrimination and Cournot competition in a vertically related market

(a) Solve for the subgame perfect Nash equilibrium values of q_1 and q_2.

We can solve the game by backward induction. Firm D1's problem is to choose q_1 so as to maximize the following profits:

$$\pi_1 = (1 - w_1 - c_1 - Q)q_1$$

The first-order condition can be written as

$$\frac{\partial \pi_1}{\partial q_1} = 1 - w_1 + c_1 - 2q_1 - q_2 = 0.$$

By symmetry, firm D2's first-order condition can be written as

$$1 - w_2 + c_2 - q_1 - 2q_2 = 0.$$

Let us here derive an equilibrium in which indeed the two downstream firms both are active, as the first-order condition above presume. For a large enough cost difference between the firms, we should expect that one of the firms chooses a zero quantity. This possibility can be studied if we want to be more ambitious with our analysis. Solving the first-order conditions for q_1 and q_2 gives us

$$q_1^*(w_1, w_2) = \frac{1 - 2(w_1 + c_1) + w_2 + c_2}{3} \qquad \text{and} \qquad q_2^*(w_1, w_2) = \frac{1 - 2(w_2 + c_2) + w_1 + c_1}{3}. \tag{4}$$

Firm U anticipates the chosen quantities in (4) and thus chooses w_1 and w_2 so as to maximize the profits $\pi^U(w_1, w_2) = w_1 q_1^*(w_1, w_2) + w_2 q_2^*(w_1, w_2)$. Taking first-order conditions w.r.t. w_1 and w_2 and then solving these for the equilibrium wholesale prices, we have

$$w_1^* = \frac{1 - c_1}{2} \qquad \text{and} \qquad w_2^* = \frac{1 - c_2}{2}. \tag{5}$$

Plugging the expressions in (5) back into (4), we get

$$q_1^*(w_1^*, w_2^*) = \frac{1 + c_2 - 2c_1}{6} \qquad \text{and} \qquad q_2^*(w_1, w_2) = \frac{1 + c_1 - 2c_2}{6}. \tag{6}$$

(b) In this new game with a ban on price discrimination, solve for the subgame perfect Nash equilibrium values of q_1 and q_2.

The analysis is very similar to the one under (a). By setting $w_1 = w_2 = w$ in the expressions in (4), we have

$$q_1^{**}(w) = \frac{1 - w - 2c_1 + c_2}{3} \qquad \text{and} \qquad q_2^{**}(w) = \frac{1 - w - 2c_2 + c_1}{3}. \tag{7}$$

At the first stage, firm U chooses w so as to maximize $\pi^U(w) = w\left[q_1^{**}(w) + q_2^{**}(w)\right]$. Taking a first-order and solving for w yields

$$w^* = \frac{2 - c_1 - c_2}{4}$$

Plugging this expression back into (7), we get

$$q_1^{**}(w^*) = \frac{2 + 5c_2 - 7c_1}{12} \qquad \text{and} \qquad q_2^{**}(w^*) = \frac{2 + 5c_1 - 7c_2}{12}. \tag{8}$$

(c) Do/answer the following:

(i) Compare the consumer surplus ($CS = [(q_1 + q_2)^2]/2$) with and without price discrimination.

Are the consumers (according to this measure) better or worse off from a ban on price discrimination?

(ii) Compare the industry profits ($\Pi = \pi_U + \pi_1 + \pi_2$) with and without price discrimination. Are the firms jointly better or worse off from a ban on price discrimination?

To answer (i), note that the consumer surplus is increasing in total output, $q_1 + q_2$. Adding up the quantities in (6), we have

$$q_1^*(w_1^*, w_2^*) + q_2^*(w_1, w_2) = \frac{1 + c_2 - 2c_1}{6} + \frac{1 + c_1 - 2c_2}{6} = \frac{1 - c_1 - c_2}{6};$$

and adding up the quantities in (8), we have

$$q_1^{**}(w^*) + q_2^{**}(w^*) = \frac{2 + 5c_2 - 7c_1}{12} + \frac{2 + 5c_1 - 7c_2}{12} = \frac{1 - c_1 - c_2}{6}.$$

That is, **total output and thus consumer surplus are the same in the two settings**.

To answer (ii), note that we can write the industry profits as

$$\begin{aligned}\Pi &= \pi_U + \pi_1 + \pi_2 \\ &= w_1 q_1 + w_2 q_2 + (2 - c_1 - w_1 - q_1 - q_2) q_1 + (2 - c_2 - w_2 - q_1 - q_2) q_2 \\ &= (2 - q_1 - q_2)(q_1 + q_2) - c_1 q_1 - c_2 q_2.\end{aligned} \tag{9}$$

That is, one component of the industry profits depends only on total output, which we showed above is the same across the two settings. The remaining component is the sum of total production costs, $c_1 q_2 + c_2 q_2$. Using the expressions in (6), we can write

$$c_1 q_1^*(w_1^*, w_2^*) + c_2 q_2^*(w_1, w_2) = c_1 \frac{1 + c_2 - 2c_1}{6} + c_2 \frac{1 + c_1 - 2c_2}{6} = \frac{c_1 + c_2 + 2c_1 c_2 - 2(c_1^2 + c_2^2)}{6}. \tag{10}$$

Using the expressions in (8), we have

$$c_1 q_1^{**}(w^*) + c_2 q_2^{**}(w^*) = c_1 \frac{2 + 5c_2 - 7c_1}{12} + c_2 \frac{2 + 5c_1 - 7c_2}{12} = \frac{2(c_1 + c_2) + 10 c_1 c_2 - 7(c_1^2 + c_2^2)}{12}. \tag{11}$$

Comparing the expressions in (10) and (11), we get

$$c_1 q_1^*(w_1^*, w_2^*) + c_2 q_2^*(w_1, w_2) \geq c_1 q_1^{**}(w^*) + c_2 q_2^{**}(w^*) \Leftrightarrow (c_1 - c_2)^2 \geq 0,$$

which always holds. That is, since industry profits are decreasing in $c_1 q_2 + c_2 q_2$, we obtain the result that **industry profits are larger with a ban on price discrimination if the firms have different cost parameters (and otherwise industry profits are the same)**.

(iii) What is the logic behind your result under (ii)? Discuss!

- We see from the analysis above that the difference in industry profits across the two settings is due to the fact that total downstream production costs are larger with price discrimination.
- One plausible explanation for why the production costs are larger is that, when price discrimination is allowed, the upstream firm effectively alleviates the cost difference between the firms when choosing w_1 and w_2 (i.e., the difference between the overall costs $c_i + w_i$ become smaller). This, in turn, means that the firm with a higher cost produces a larger share of industry profits under price discrimination than without—leading to higher total downstream production costs.
- The analysis above did not take into account the possibility that one firm is not active and that the likelihood that this happens might differ across the two settings (price discrimination or not). This is another natural discussion point.